

Math Matters

How the Common Core Will Help the United States
Bring Up Its Grade on Mathematics Education

By Max Marchitello and Catherine Brown

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Introduction and summary

The Washington state technology industry is booming. The state is home to giant tech corporations such as Microsoft and Amazon, and it has the highest concentration of software companies in the country.¹ This prosperity is expected to continue and estimated to increase the state's science, technology, engineering, and math, or STEM, economy 24 percent by 2018.²

A similar trend is expected nationwide. Today, 20 percent of all jobs across the country require a high level of knowledge in a STEM field.³ Experts predict that these fields will be among the country's highest-growth industries in the years to come.⁴

Less well known is that knowledge of STEM fields is important not only for highly educated workers, but also for those without a college degree. According to the Brookings Institution, “Half of all STEM jobs are available to workers without a four-year college degree, and these jobs pay \$53,000 on average—a wage 10 percent higher than jobs with similar educational requirements.”⁵ Even “blue-collar or technical jobs in fields such as construction and production ... frequently demand STEM knowledge.”⁶ Moreover, employers report that “mathematical knowledge will be either very important or extremely important to success” in 70 percent of jobs.⁷ In other words, the demand and reward for workers who are skilled in critical thinking and problem-solving is rising, while the number of opportunities available to workers without these skills continues to decline.⁸

Unfortunately, the U.S. public K-12 education system is not preparing all students to seize opportunities for challenging, well-paying jobs. Recent projections warn that if current trends continue, there will be 5 million unfilled jobs by 2020—largely because prospective workers lack the requisite skills.⁹ A recent survey of employers by the Association of American Colleges & Universities found that the majority of employers do not feel that “recent college graduates are well prepared,” particularly when it comes to critical thinking skills.¹⁰

Meanwhile, other countries do a far better job of readying their students to be competitive in today's job market. According to the most recent data from the Programme for the International Assessment of Adult Competencies, or PIAAC, adults in the United States have “very poor” numeracy skills.¹¹ In fact, out of 24 countries, American adults scored higher than only two, and nearly one-third of adults demonstrated below basic mathematics skills.¹² The opposite is true in other countries. Approximately 20 percent of adults scored at PIAAC's highest levels in Japan and Finland, compared with less than 9 percent in the United States.¹³

American high school students also perform far below the international average in math.¹⁴ Currently, they rank 27th in mathematics, while Korean and Japanese students lead the world. Between 2003 and 2012, the average math score in the United States actually decreased 2 points, while Korea's shot up 12 points.¹⁵ Even the most affluent American students scored significantly below the average score of other countries. For example, American students in the highest economic quartile scored 81 points lower than the average student in Shanghai, China.¹⁶

The Common Core State Standards—which 43 states and the District of Columbia currently use¹⁷—were developed to address these problems. Through the Common Core, students are taught to understand both the procedures for doing math problems—such as memorizing multiplication tables or learning to “carry the 1”—and how and why these procedures work. This approach allows students to more deeply understand the concepts that underlie mathematics, improve their critical thinking skills, approach problems from different perspectives, and apply what they learn to real-world problems. In other words, students will study why $5 \times 10 = 50$ in addition to memorizing multiplication tables.

The Common Core math standards represent the culmination of decades of research into how students learn and are an extension of 30 years of standards and curriculum development by the National Council of Teachers of Mathematics, or NCTM.¹⁸ Ultimately, learning math this way helps students progress through advanced math while in school and better prepares them for a wide range of careers, particularly those in high-growth STEM fields.

As with any shift in pedagogy and practice, however, it will take time for educators, parents, and students to become accustomed to this approach to mathematics. This being said, patience and faithful implementation of the standards will pay dividends. Students in states that adopted the Common Core early, such as Kentucky, have begun to make important progress. According to a study of student performance on the ACT—a common measure of college readiness—

conducted one year into the implementation of the Common Core and again two years later, students in Kentucky made gains that amount to an additional three months of learning.¹⁹ The key is staying the course, providing high-quality and continuous professional development and support to teachers, and routinely communicating with parents about what their children are learning and how it prepares them for college and careers.

The Center for American Progress has reviewed the literature and research and discovered several reasons why teaching for conceptual understanding of math leads to improved outcomes for students:

- The traditional method of mathematics instruction has focused on rote memorization and procedures that have not led to a deep understanding of the concepts that underlie math. Without a deep understanding of mathematical concepts, American students struggle to problem solve effectively and to apply their skills to real-world situations.
- Students who are taught to conceptually understand math outperform students who are taught to use the traditional algorithmic approach.
- In order to achieve mathematical fluency, students must learn to make connections and draw conclusions from new material. Conceptual math allows them to do this.

The Common Core adds conceptual math to the traditional procedural way math is taught, allowing students to gain both mathematical fluency and skill proficiency.

In order to transition to conceptual math and to ensure that students, teachers, and parents are comfortable and prepared for success, CAP makes the following recommendations:

- **States should stay the course with Common Core standards and aligned assessments.** Through perseverance, the nation can improve the quality of mathematics education for all students.
- **States and districts should provide teachers with additional, dedicated time and professional development opportunities.** High-quality, ongoing, and readily available professional development allows teachers to internalize the standards with the help of effective instructors. States should also develop a standards translation guide for teachers.
- **Districts should communicate regularly with parents and provide training and resources.** This helps parents support their children as they learn conceptual math.

- **States and districts should review curricula and instructional materials to ensure that they are high quality and aligned with and supportive of Common Core math.** By doing this, schools can ensure that students are being taught effectively.
- **Teacher preparation programs should incorporate conceptual mathematics into curricula.** Future teachers should also be trained to instruct students in conceptual math.

Shifting to math education of this caliber and depth is difficult and will likely challenge both teachers and students. States must support educators as they become fluent in and adapt their practices to ensure that students engage meaningfully with math and learn to think beyond simple formulas and processes. Otherwise, American students' math performance will continue to slip below what the global economy requires.

The state of math education in the United States

Although math achievement has improved and race- and class-based achievement gaps in math scores have narrowed over the past decade, U.S. math performance remains low and large gaps persist.²⁰ Over the same time period, the performance of students in many other countries has soared.

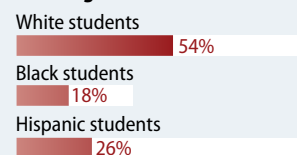
According to the National Assessment of Educational Progress, or NAEP, often referred to as “The Nation’s Report Card,”²¹ less than half of the nation’s fourth-grade students and slightly more than one-third of its eighth-grade students are on track to graduate college and career ready.²² These figures are alarming, but the state-by-state variation is even more startling. Minnesota and New Hampshire lead the nation with 59 percent of their fourth-graders achieving proficiency,²³ while only 26 percent of students reach the minimum bar of proficiency in Mississippi and Louisiana.²⁴ Since misconceptions about numerical values and mathematical relationships are compounded as students move to more challenging math lessons, it is unsurprising that student performance is worse in eighth grade than in fourth grade. In Massachusetts—the highest performing state—55 percent of eighth-grade students are proficient, compared with only 19 percent in the District of Columbia and 20 percent in Alabama.²⁵

What do these proficiency rates mean in terms of students’ actual knowledge once they graduate high school? To answer that question, researchers in 2010 analyzed the performance of community college students in developmental math classes across a variety of mathematical domains.²⁷ They theorized that how well community college students perform in math reveals something about their K-12 math experiences.²⁸

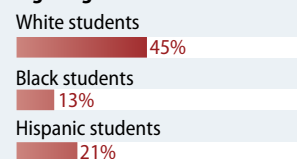
What they found was troubling: The majority of students could not accurately place fractions or decimals on a number line. Most of the students were unable to recognize patterns and relationships between values and operations. For example, only 21 percent of students successfully placed -0.7 and $\frac{13}{8}$ on a number line. In a similar question, the researchers asked, “If a is a positive whole number, which is greater:

Even with overall math performance at an unacceptably low level, there are pernicious, large achievement gaps between white students and students of color.²⁶ Improving the quality of math education is particularly critical for students of color.

Fourth-grade math



Eighth-grade math



a/5 or a/8?” Only 53 percent of students chose correctly. When asked to explain their reasoning, only 15 percent of students supported their choice with a conceptual explanation.²⁹ Notably, each of those students answered the question correctly.³⁰ The authors argue that this, among other findings of students’ poor grasp of mathematics, is the consequence of focusing math education on procedures, tricks, and step-by-step processes in lieu of concepts.³¹

These findings are consistent with the conclusions of the 2008 National Mathematics Advisory Panel.³² In their report, the authors argue that “difficulty with fractions ... is pervasive and is a major obstacle to further progress in mathematics.”³³ In particular, they point to the poorly understood relationship between a procedural knowledge of fractions and their conceptual underpinnings.

Poor student performance on the NAEP and with fractions speaks to a broader problem: Many American students have a weak understanding of underlying mathematical concepts. They know how to solve math problems, but they do not understand why their solutions work or know how to apply mathematical concepts and reasoning to other situations. Many students struggle with fractions because they do not understand their numerical values or their relationship to whole numbers. Because they cannot grasp these fundamental concepts, students cannot apply logical reasoning to more challenging problems. In short, their critical thinking skills in mathematics are limited.

Students from other countries outperform American students

The gap between the performance of American students and those in other developed countries is gaping—and growing. In the most recent round of tests from the Programme for International Student Assessment, or PISA, 15-year-old American students scored 13 points below the international average and 92 points below Singapore.³⁴ Overall, students in the United States performed in mathematics more or less the same as they did in 2003, while students in other countries improved considerably. For example, Poland jumped from an average score of 490 in 2003—higher than the United States scored—to 518 in 2012.³⁵

In addition, many U.S. students fail to demonstrate a basic level of mathematics knowledge and skill. On the most recent test in 2012, more than 25 percent of American students did not reach PISA’s baseline level 2, which corresponds with the “skills that will enable them to participate effectively and productively in life.”³⁶

At the same time, the percentage of students who failed to meet this minimum threshold in Singapore and South Korea is closer to 10 percent. Rather than competing with world-leading countries, American students' performance more closely resembles that of Lithuania and Luxembourg, which are far from being world leaders in education.³⁷

At the most advanced levels, American students are still behind. Only 2.2 percent of American students reach PISA's highest level of performance, compared with more than 30 percent in Shanghai, China.³⁸ Even students in one of the highest-performing states in the country, Massachusetts, score comparably low. In fact, the average scores of students in Shanghai suggest that they are more than two years ahead of students in Massachusetts.³⁹

American students perform marginally better on the Trends in International Mathematics and Science Study, or TIMSS.⁴⁰ Given how math has been taught in the United States, it makes sense that American students would be more successful on TIMSS, which is "focused on formal mathematics knowledge, whereas PISA emphasizes the applications of mathematics in the real world."⁴¹ In 2007, the average mathematics TIMSS score for U.S. fourth- and eighth-graders was higher than the international average. In fact, in fourth- and eighth-grade TIMSS scores, only eight countries and five countries, respectively, outperformed American students.⁴² Yet the TIMSS data also reveal race- and class-based achievement gaps similar to those found in the NAEP data. For example, black students score 51 points below the national average for the United States. If the performance of black American students was ranked among that of the participating countries, it would land at number 27, just between Romania and Bosnia and Herzegovina, and 18 spots below the United States overall.

In sum, American students—particularly low-income students and students of color—struggle with more cognitively demanding tasks, such as "taking real-world situations, translating them into mathematical terms, and interpreting mathematical aspects in real-world-problems."⁴³ American students are stronger at more basic tasks, such as "extracting single values from diagrams or handling well-structured formulae."⁴⁴ In other words, they have a weak conceptual understanding of mathematics.

Today's jobs require greater knowledge and skills

Not too long ago, a poor understanding of mathematics did not harm one's ability to find a good job and to enter the middle class. That is not the case today. At a time when calculators and computers are ubiquitous, workers need to move beyond the surface to understand underlying concepts, be deep thinkers and problem solvers, and apply their skills to new situations. Beyond the need for adults to understand and navigate contemporary technology and the ever-changing economy, employers also expect more from potential employees. A recent IBM survey of more than 1,700 CEOs from around the world found that analytical and quantitative skills are among the traits that employers most value.⁴⁵

Even students who do not plan to go to college need to have a strong conceptual understanding of mathematics and be critical thinkers. The Georgetown University Center on Education and the Workforce estimates that 96 percent of future jobs will require workers to be critical thinkers and that 70 percent will require employees to have strong mathematics skills.⁴⁷ Simply put, being good at math is no longer just for the college bound. All students need to be grounded in conceptual math in order to compete for the growing number of technical and computer-related jobs—many of which are in high-growth careers that pay high salaries.⁴⁸

In light of these sobering statistics, it is becoming increasingly clear that states need to improve how math is taught. The problem is not that the traditional way of teaching students to solve $15 + 23$ does not lead to the correct solution of 38. Rather, it does not fully teach students the conceptual basis of addition. Ultimately, students need to know how math works and be able to apply what they know in order to meet the demands of today's economy.

"As a nation, we must unite in recognizing the mounting evidence that the U.S. is falling behind international competitors in producing students ready for 21st-century jobs."⁴⁶

— Rex Tillerson, Chairman and CEO, Exxon Mobil Corp.
September 5, 2013

The promise of conceptual mathematics

The shift to the Common Core and embedding conceptual math into mathematics standards alongside the more traditional instructional approach to mathematics helps improve students' critical thinking and problem-solving skills.

Conceptual math: The history and research basis

Although it has been labeled as “new,”⁴⁹ teaching the conceptual basis of math is a decades-old American idea that has revolutionized mathematics education—but not in this country.⁵⁰ In the 1980s, the National Council of Teachers of Mathematics attempted to incorporate these research findings into mainstream K-12 education.⁵¹ They, as well as other experts and practitioners, advocated for classroom activities that shift away from teaching step-by-step processes and instead toward teaching for greater conceptual understanding of math. Other countries, such as Japan, adopted these new methods to great success.⁵²

Research over time has documented their power. An Organisation for Economic Co-operation and Development, or OECD, study of the similarities between PISA and the Common Core “suggests that a successful implementation of the Common Core Standards would yield significant performance gains.”⁵³

Other research shows that students who are exposed to conceptual approaches to mathematics consistently outperform their peers on more complicated mathematics problems.⁵⁴ And some studies found that instruction based on a conceptual understanding of math prepared students to outshine their peers even in skill efficiency.⁵⁵ In other words, conceptual math not only improves students' mathematical reasoning and problem-solving skills, but also improves their ability to use mathematical processes and procedures.⁵⁶

Early studies of the impact of different mathematics instruction on student achievement documented the positive effects of learning math conceptually. In 1949, education psychologists William Brownell and Harold Moser studied the comparative effects of teaching subtraction “meaningfully,” using physical objects that represent base-10 groups—or a value of 10—and “mechanically,” through the traditional algorithm.⁵⁷ After six weeks, the conceptual approach to subtraction resulted in higher scores, particularly in retention and skills transfer.⁵⁸ Overall, the authors concluded that the meaningful approach showed the “most promise”⁵⁹ to help students learn to think critically and to transfer their conceptual understanding to new problems.

A highly controlled 1980s study on the effect of conceptual math instruction on student performance had similar findings. In this study, some fourth-, sixth-, and eighth-grade teachers were asked to teach using very structured lesson plans based on conceptual mathematics, while others were asked to use their own method. Based on pre- and post-tests, the students taught by the conceptual math lesson plans improved significantly more than their peers. In fourth grade, for example, student performance jumped from the 27th percentile to the 57th percentile on a nationally normed assessment, significantly higher than that of their peers.⁶⁰

The positive effect of conceptual math on student performance is supported by many other studies, including a 1990 analysis of the impact of a conceptual approach to addition and subtraction on student learning outcomes.⁶¹ In this study, first- and second-graders in Chicago received specific addition and subtraction instruction using base-10 blocks to compose and decompose numbers that gradually increased in value.⁶² Before learning this approach, all students were taught the algorithmic approach to addition and subtraction. Based on this instructional method, fewer than 10 percent of students could correctly use the blocks to represent the computations on a pretest. After learning to add and subtract using base-10 blocks and gaining a conceptual understanding of the value of three-digit numbers, 160 of the 169 students demonstrated the ability to compose and decompose up to four-digit numbers effectively.⁶³

In order to be able to manipulate the base-10 blocks, students must fully understand value transfer, place value, and the relationship between different numbers. This study demonstrates that the algorithmic method does not teach students these important concepts. After using the blocks, however, students had a much more complete and conceptual understanding of these ideas and were far more successful at composing and decomposing large numbers.

Conceptual math often requires students to grapple with new concepts and procedures before a teacher formally instructs them. James Hiebert and Douglas Grouws found this “struggle” to be consistently important in improving student outcomes but conspicuously missing from American classrooms.⁶⁴ Specifically, they argue that to successfully gain a conceptual understanding of mathematics, students must “expend effort to make sense of mathematics, to figure something out that is not immediately apparent.”⁶⁵ The authors highlight research and writing by theorists ranging from John Dewey⁶⁶ to Giyoo Hatano,⁶⁷ who developed “the notion of cognitive perplexity as a central impetus for cognitive growth.”⁶⁸ Beyond helping students apply their prior knowledge to problem solve and develop their analytical skills, struggling with more complex questions teaches students to persevere and gives them confidence in their own knowledge and skills, characteristics that will serve them later in life.⁶⁹

Incorporating conceptual mathematics into everyday math instruction plays a crucial role in improving student performance. Learning the foundations of mathematical concepts and operations provides students with the building blocks they need to construct solutions to more complex and cognitively demanding tasks. As a result, students become stronger critical thinkers and problem solvers and will be better prepared for the rigor of today’s job market.

Procedural versus conceptual math instruction

Historically, math instruction in the United States has focused on teaching students strategies and tricks to quickly ascertain the correct answer rather than understanding what they have learned and applying the concepts to other scenarios. This model largely worked to prepare most students for the jobs and economy of yesterday.⁷⁰ But students need to have a deeper understanding of mathematics if they hope to access the more demanding jobs of today.

The Common Core answers this call by providing a new way for teachers to teach and students to learn math. The standards incorporate two key practice shifts. First, there are fewer, more-comprehensive fluency-based standards. This narrowed focus allows teachers more time to teach the underlying mathematical concepts and provide students with ample opportunity to engage deeply with the subject. Furthermore, in order to meet the expectations of the standards, students need to spend far more time working through math problems that require them to make connections between concepts and apply their knowledge to new situations—precisely the kind of work that should be included in American math classes.⁷¹

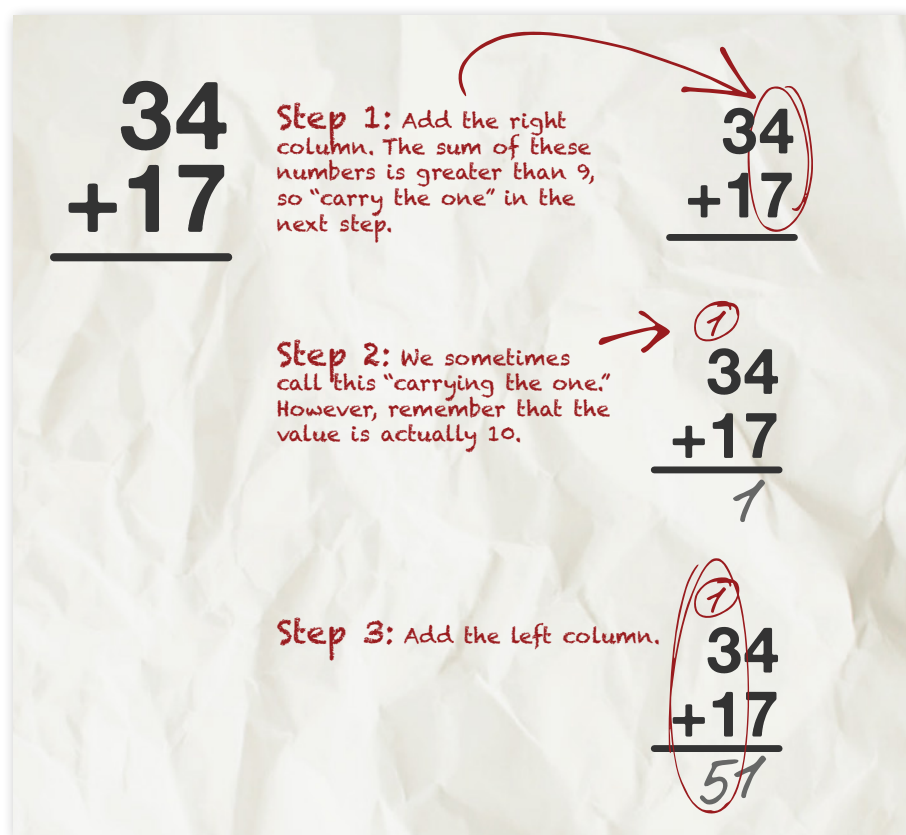
This approach differs from previous standards, which required a shallower mastery of many more concepts. The first-grade standards, for example, now include only four critical domains, each of which represents a major area of study: operations and algebraic thinking; numbers and operations in base 10; measurement and data; and geometry.⁷² Within each domain, there are between three and eight standards that describe more specific skills and knowledge. As previously noted, however, these standards are more complex and require far more time to master than most previous state standards.

The Common Core standards embed conceptual math directly into the curricula alongside the traditional algorithmic approach. In order to meet the standards, students need to be taught to add and subtract increasingly large numbers and use representations of value, such as a bushel of 10 apples, to solve problems. Students must also fully grasp numerical value.

Teaching students to understand concepts and apply them to other problems is different than teaching them to solve problems as efficiently as possible. The text box below presents some of the key differences.

Conceptual knowledge versus procedural knowledge ⁷³	
Conceptual knowledge	Procedural knowledge
▼	▼
Knowledge rich in relationships and understanding	Knowledge of formal language or symbolic representations
Examples of concepts: Square, square root, function, area, division, linear equation, derivative, and polyhedron	Knowledge of rules, algorithms, and procedures
Learned through thoughtful, reflective mental activity	Learned through rote memorization

To illustrate this difference with a real math problem, consider how U.S. schools have typically taught students to add two-digit numbers:



In order to solve this problem, students were taught to follow a simple process called "carrying the one." When the first column on the right, or ones place, adds up to a number larger than nine, a one is put on top of the digit to the left and included in the calculation for the next column. This is the standard algorithmic or procedural approach to addition.

This method misrepresents the actual value and relationship between these two numbers. To see this, one needs to look no further than the trick of carrying the one: It actually adds 10. In favor of easily finding the correct solution, this strategy actually misleads students about the relationship between the numbers. Unfortunately, there is also a similar practice in subtraction.

In contrast, to teach a conceptual understanding of the relationship between the added numbers in the example above, a teacher might approach the problem like this:

The image shows a piece of crumpled paper with three math problems written in black ink. Red handwritten annotations in parentheses with arrows explain the place value components of the numbers:

- For the first problem, $30 + 10 = 40$, an arrow points from "(three 10s)" to the 30, and another arrow points from "(one 10)" to the 10. The result 40 is annotated with "(four 10s)".
- For the second problem, $4 + 7 = 11$, the result 11 is annotated with "(one 10 and one)".
- For the third problem, $40 + 11 = 51$, the result 51 is annotated with "(five 10s and one)".

Another way that students may be asked to demonstrate their conceptual understanding of place value and numerical relationships is to illustrate sum groups of 10s and ones on a number line. This helps students by breaking large numbers into their composite parts and into sums, such as 10s, that are easier to manipulate. This practice is important for students to learn problems' numerical values and the relationships between numbers. For example, students could be asked to identify 54 on a number line and compose that sum through 10s and 1s.

Similarly, many students who have a procedural but not a conceptual understanding of mathematics will insist that 1.15 is bigger than 1.5 because 15 is greater than 5. This common mistake reveals that students do not understand the value of whole numbers.

Many students will also answer that one-sixth is more than one-third because six is greater than three. In reality, correctly identifying the larger number requires understanding how numbers relate to one another.

Teaching rote processes rather than concepts has been commonplace even in more-advanced classes, such as calculus. Under the procedural method, for example, students are taught a two-step process to take the derivative of a polynomial equation—a mathematical function comprised of variables and exponents.

Calculus in two steps

For many students, completing calculus is the ultimate math achievement. In order to be successful in calculus, students must be able to take the derivative of a function. Solving the derivative allows students to determine the instantaneous velocity or rate of change at any given point, no matter how small. Simply put, students must fully grasp derivatives in order to even begin to understand the physics of motion.

Yet how to find a derivative can be taught through a simple process without ever unpacking what a derivative is or its relationship with other mathematical concepts. To take the derivative of the following function, for example, students need only take two short steps:

$f(x) = 4x^3 + 7x^2$

Step 1: Multiply each coefficient, $4x$ and $2x$, by the value of its exponent.

$3 \cdot 4x^{(3-1)} + 2 \cdot 7x^{(2-1)}$

Step 2: Subtract one from the value of each exponent.

Answer: $f'(x) = 12x^2 + 14x$

This is the procedural approach to taking a derivative. However, derivatives are complex mathematical functions. Conceptually, a derivative measures change. More specifically, it is function that specifies the

rate of change of another function. For example, take a function that describes the arc of a baseball after a player hits it. The derivative allows you to solve for the speed of the baseball at any time.

Examples of middle school math questions

Students need to be able to do arithmetic in order to solve both of the problems below. In the traditional math example, students are asked to use arithmetic to solve an arithmetic problem. But in the Common Core question, students are asked to take their understanding of addition, subtraction, and multiplication to the next level. In order to solve the problem, they must apply those concepts to a more complicated and involved question. In other words, this problem demands that students take those same arithmetic concepts but employ them to solve an algebra problem. This type of critical thinking is made possible through students’ strong conceptual understanding of mathematics.

	Traditional math	Common Core math
Question	Donna buys 40 apples at 35 cents each. She eats 2 apples and sells the rest for 45 cents each. How much money does she make?	Donna buys some apples at 35 cents each. She eats 2 apples and sells the rest for 45 cents each. She makes \$4.40. How many apples did she buy?
Method	Simple arithmetic	Algebraic equation
Process	$40 \times \$0.35 = \14.00 $(40 - 2) \times \$0.45 = \17.10 $\$17.10 - \$14.00 = \$3.10$	$0.45(x - 2) - 0.35x = \$4.40$ $0.45x - 0.9 - 0.35x = \$4.40$ $.10x - 0.9 = \$4.40$ $.10x = \$3.50$ $x = 35$
Solution	Donna made \$3.10.	Donna bought 35 apples.

Source: Foundation for Excellence in Education, “Old Standards v. Common Core: A Side-by-Side Comparison of Math Expectations,” available at <http://excellent.org/common-core-toolkit/old-standards-v-common-core-a-side-by-side-comparison-of-math-expectations/> (last accessed May 2015).

The Common Core also exposes students to the conceptual basis of complex mathematics, including geometry and fractions, at an early age. As early as first grade, the standards introduce such concepts and lay the conceptual foundation necessary for students to better understand the relationship between shapes, as well as between individual parts and larger numbers.

Consider the following Common Core teacher standard:⁷⁴

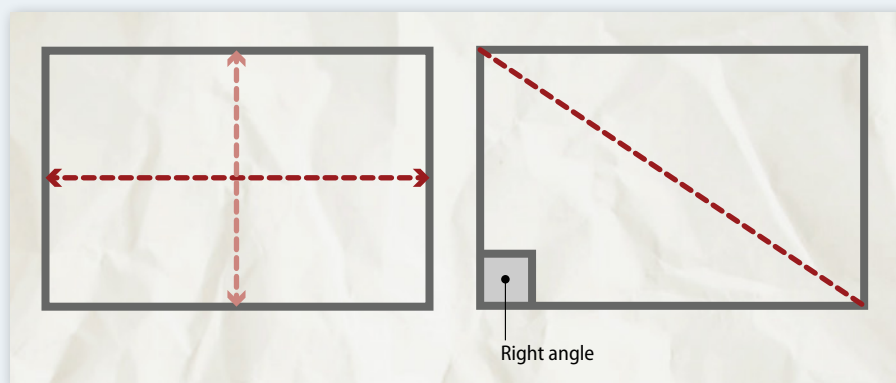
Reason with shapes and their attributes.
Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Here, students learn several vital and related concepts. They begin to navigate geometry and are exposed at an early age to the underlying principles necessary to understand geometric area and perimeter. Furthermore, by dividing a whole into equal parts, students, perhaps unwittingly, create fractions—showing, for example, that a piece is one-fourth of a rectangle, or $\frac{1}{4}$.

Learning the foundations of geometry in first grade

From a child's earliest years playing with blocks to growing up to become a world-renowned architect, geometry is ever present, underscoring the fact that students need to learn to reason and think critically—not just about numbers, but also about spaces, shapes, and angles. The Common Core embeds the principles of geometry into the standards as early as first grade—long before a student

learns trigonometry. Consider a rectangle. There are two ways to cut it into two equal parts that quickly jump to mind: horizontally and vertically. However, after some time, a student likely would discover another way to bisect the rectangle: diagonally. This approach uses the hypotenuse of a right-angle triangle.



This realization illustrates to students that rectangles can also be thought of as two equal, right-angle triangles. Students can then think about rectangles as a combination of two triangles. Not long after, they might figure out that they can draw another line and create four triangles. In this instance, however, the triangles do not have a right angle nor are they all equal. Why? As students build their spatial understanding, they are able to reason through this question. Mastering this concept is important as students begin to study the area of different objects.

In a typical classroom, for example, teachers simply tell students the formula for the area of a right triangle. But with mastery of the relationship between triangles and rectangles, the formula and its conceptual basis become clear—simply find the area of a rectangle and divide by two. Or as most were likely taught: $(\text{base} \times \text{height})/2$.⁷⁵ Forging these kinds of connections between seemingly disparate concepts teaches students to look for relationships and patterns and to think critically about problems. These skills are vital if students want to progress through school and into college and careers.

In truth, fractions are the first true test of a student's numeracy skills. Without a strong grasp of value, students misunderstand how fractions relate to other fractions or how a fraction, such as five-fourths, relates to a whole number, such as two. By learning in first grade that a single square can be composed of four smaller, equal parts, students can later jump to understanding more complex fractions or even fractions that are greater than one whole. To build this understanding, the Common Core delays combining fractions with mathematical applications such as addition and multiplication in order to ensure that students first fully grasp the values that fractions represent.⁷⁶

Consider the following third-grade math standards:⁷⁷

Develop understanding of fractions as numbers.

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.

This standard—and in particular, its substandard—get to the heart of a common misconception students have about fractions: They do not have to represent a value that is less than one. This misunderstanding is reasonable given that fractions are almost always introduced as slices of pie or pieces of a whole. This false conception is typically reinforced by the idea of so-called improper fractions, or fractions that represent values greater than one, such as five-fourths. The truth is that these fractions are not actually improper by any means but rather another way to represent numerical value and explain relationships between numbers. Understanding these concepts is vital to students' success in more complicated manipulations of fractions.

The Common Core does not rely solely on conceptual mathematics, however. The traditional algorithmic method of math that most adults learned remains a large part of the Common Core. After learning the concepts of addition, students learn how to use the process of carrying the one to solve equations quickly. The same is true in multiplication and other practices. Learning the conceptual approach to math is just as important as learning to add with paper and pencil before using a calculator.

In fourth grade, for example, students need to be able to skillfully apply the procedural, algorithmic approach to add and subtract:⁷⁸

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

The same is true in multiplication. In fifth grade, students need to fluently use the standard multiplication algorithm after learning the conceptual foundations of multiplication in fourth grade:⁷⁹

Perform operations with multi-digit whole numbers and with decimals to hundredths.

Fluently multiply multi-digit whole numbers using the standard algorithm.

The Common Core standards are constructed so that the math concepts students learn build on each other. For example, how a first-grader learns to use the number 10 to construct larger sums and then to break apart bigger numbers into their smaller number of 10s helps set the conceptual basis for multiplication as a series of additions before students even hear the word “multiplication.” The standards also incorporate the traditional algorithmic approach to math, preserving its important function in helping students practice mathematical procedures after they have developed a conceptual understanding of what they are actually doing. This model helps improve fluency and skill efficiency and, ultimately, better prepares students for college and careers.

The ultimate goal for mathematics instruction must be to enable students to apply mathematical concepts to real-world problems and figure out the best way to solve them. In other words, students need to learn to be critical thinkers and problem solvers—the very skills they will need to be successful in their professional lives.

Example of assessing procedural versus conceptual understanding of mathematics

The following are three different ways to pose the same question. But the knowledge and skills students need to correctly solve the question depend on whether the problem provides a formula and answer choices. The first two problems require only procedural knowledge, while a conceptual understanding of algebra and fundamental physical relationships is needed to solve the final question.

The following problems are adapted from Sendhil Revuluri and Mary Jo Tavormina, Deepening Understanding of Mathematics Assessments of and for Learning (Chicago: University of Illinois, 2015). Solutions and commentary offered by the authors.

Consider the following assessment tasks:

- What kind of thinking and reasoning does each task demand?
- What inferences could teachers make about student understanding?
- How could the evidence inform or guide teachers in taking next steps?

Understand solving equations as a process of reasoning and explain the reasoning...
Solve simple rational ... equations in one variable... (CCS SMC HS.A-REI.2)

Item A.1

A plane flies at an air speed of 450 miles per hour, but its ground speed varies because of the force of wind. It can travel 980 miles when going in the direction *with* the wind in the same amount of time that it takes to travel 820 miles going *against* the wind. Find the speed of the wind.

Solve the equation

$$\frac{980}{450 + s} = \frac{820}{450 - s}$$

to find the speed s of the wind.

- A. 20 mph
- B. 30 mph
- C. 40 mph
- D. 50 mph
- E. 60 mph

The solution:

C. This is the easiest of the problems. Here, students can simply plug in each answer until they find one that balances the equation. In the correct solution, students substitute 40 for “ s ” and get $980/490 = 820/410$. Students can quickly see that each denominator is half the numerator. Therefore, both are equal to two. Students could also divide each side of the equation to see if they are equal, or they could cross multiply.

Item A.2

A plane flies at an air speed of 450 miles per hour, but its ground speed varies because of the force of wind. It can travel 980 miles when going in the direction *with* the wind in the same amount of time that it takes to travel 820 miles going *against* the wind. Find the speed of the wind.

Solve the equation

$$\frac{980}{450 + s} = \frac{820}{450 - s}$$

to find the speed s of the wind.

The solution:

$s = 40$. This problem is somewhat more difficult, since students need to demonstrate an ability to cross multiply with a variable and then simplify an algebraic expression.

Step 1: Cross multiply and get $980(450 - s) = 820(450 + s)$.

Step 2: Distribute 980 and 820 to get $441,000 - 980s = 369,000 + 850s$.

Step 3: Combine like terms to get $72,000 = 1,800s$.

Step 4: Solve for “ s ”, with s equaling $72,000/1,800$.

Item A.3

A plane flies at an air speed of 450 miles per hour, but its ground speed varies because of the force of wind. It can travel 980 miles when going *with* the wind in the same amount of time that it takes to travel 820 miles going *against* the wind. Find the speed of the wind.

The solution:

The wind travels at 40 mph. This is the most challenging problem. Here students are required to use the information available to develop their own equation to solve the problem. Students must realize several things in order to solve this problem:

1. The plane's ground speed is determined by the plane's air speed.
2. The wind speed is constant, but its effect on the plane's speed depends on its direction. When going with the wind, the plane's ground speed is increased by s , or $(450 + s)$. When going against the wind, the plane's ground speed is decreased by s , or $(450 - s)$.
3. Distance = speed \times time.
4. The problem says that trips in both directions take the "same amount of time."

This means that time = time. So distance in one direction/speed in that direction = distance in other direction/speed in that direction. In other words:

$$\frac{980}{450 + s} = \frac{820}{450 - s}$$

How to make the shift to conceptual math successful

There is a considerable body of research suggesting that teaching conceptual math improves students' numeracy and critical thinking skills and better prepares them for college and careers. Nevertheless, these seemingly new approaches to mathematics can appear quite strange⁸⁰ to many parents, even those who excelled in math when they were in school. Their reactions are understandable: Math instruction has evolved under the Common Core. But just as parents want the best for their children in all other respects, they should want teachers to use the latest evidence-based instructional practices. Breakthroughs in medicine over the past two decades enable today's surgeons to remove brain tumors with minimally invasive surgery that uses a small tube inserted in a patient's nose. Even AIDS has been turned into a chronic but largely manageable condition in the United States through the invention of the drug AZT.⁸¹

Experts' understanding of how children learn to read has evolved too. A clear consensus has emerged about the need to teach phonemic awareness, phonics instruction, and more when teaching children to read.⁸² Angela Duckworth, Carol Dweck, and others have galvanized a focus on self-regulation skills, showing them to be teachable and associated with better life outcomes for all students.⁸³ Research in cognitive science has led to breakthroughs in understanding how children's brains develop during different stages of childhood and when they are living through poverty or trauma; these breakthroughs have resulted in new teaching techniques.⁸⁴

Teaching conceptual math requires educators to spend more time ensuring that all of their students have a deep understanding of the underlying mathematical concepts and relationships between those ideas before introducing the traditional algorithmic methods of calculation. To do this, teachers need to change not just what they teach but also how they teach it.

As such, it may be necessary to restructure how material is introduced to students. For many adults, math class followed a familiar pattern: I do, we do, you do. In this approach, teachers first model a new practice, students work together on a similar set of problems, and the students then finish the work individually. In order to teach conceptual mathematics and help students apply their prior knowledge to solve new

problems, some education theorists argue that the traditional structure should be inverted. In other words, students should first use what they already know to grapple with new concepts before working with their peers and, finally, with their teacher.⁸⁵

For this strategy to work, students must have a firm handle on the conceptual basis of mathematics so that they can, for example, use their understanding of addition to solve multiplication problems. This approach to problem-solving has several benefits. First, it fosters creative and critical thinking in students by providing them with knowledge and skills they can bring to bear on new problems and ideas. Second, it allows students to find multiple pathways to solutions, generating a deeper understanding of the new concept and how what is already known relates to what is being taught now. All in all, teaching students conceptual math—and providing them with space to use their prior knowledge to solve new problems—is a good thing and helps improve student math performance.

To successfully improve mathematics education, however, teachers cannot be expected to go it alone. States and school districts must make considerable investments to ensure that their curricula are aligned with standards and that all instructional materials are high quality and help students meet the standards. Furthermore, teachers must have opportunities for robust, embedded, and ongoing professional development and significant time with effective instructors to help them internalize the standards and new approaches to teaching, lesson construction, and analysis of student work.

The success of the Common Core and conceptual mathematics hinges in part on the development of aligned and useful instructional materials. Unfortunately, there are many examples of low-quality math problems that confound students and parents alike. Recent alignment studies that compare math textbooks and instructional materials with the Common Core have found that many are misaligned with the standards.⁸⁶ This is an urgent problem. States and districts must invest in developing high-quality materials aligned with curricula and standards so that teachers are well prepared and supported to teach students more challenging mathematics.

These changes will not happen overnight. On the contrary, successfully implementing conceptual math and realizing the goal of the Common Core—to ensure that all students are ready for college and careers—will be a long, iterative process. Fortunately, with the vast majority of states working toward the same goal, there are myriad opportunities for collaboration and sharing of best practices. Together, these efforts ultimately help students develop the strong critical thinking, problem-solving, and mathematical skills they will need to be successful in the 21st-century workforce.

Recommendations

Learning and teaching conceptual math is not easy. The Common Core asks students and teachers to undertake significant academic and practical shifts in how they think about and do mathematics. These changes, although challenging, represent the very best educational methods for students to learn math. After learning conceptual math, students will be stronger critical thinkers and problem solvers, as well as better prepared for the rigor of today's workforce. That being said, there are ways that states and districts can ease the transition to conceptual math and support students and teachers. Based on best practices from across the country, the Center for American Progress makes the following recommendations:

States should stay the course with Common Core standards and aligned assessments

Common Core math standards are internationally benchmarked, comparably as rigorous as the standards used by high-performing nations, and grounded in research about the skills that students need to succeed in today's economy. While the transition has been challenging for teachers, parents, school leaders, and students—as any major transition is—all key stakeholders should take the time to learn the new standards before passing judgment on them. Decades of research demonstrate that the United States can do better when it comes to math achievement. The Common Core State Standards offer a path forward.

A recent study by the American Institutes for Research found that students in Kentucky, the earliest adopter of the standards, have made more progress in college readiness than they did with the state's previous standards.⁸⁷ Although the study's authors cannot necessarily attribute all of the progress to adopting the Common Core, they nevertheless found that students “with more exposure to the standards ‘made faster progress in learning’ than peers who followed the older state standards.”⁸⁸

These promising results are the product not only of raising standards but also of consistent and considerable investments by the state and school districts to ensure teachers are adequately prepared to teach the Common Core. In addition, this study highlighted what most teachers already know: Curricula and instructional materials matter. Student performance in subjects with curricula aligned with the Common Core saw “larger, more immediate improvement than student performance in subjects that carried over last-generation curriculum framework.”⁸⁹

Common Core-aligned assessments—such as the Partnership for Assessment of Readiness for College and Careers, or PARCC, and Smarter Balanced—are the highest-quality tests of whether students have mastered Common Core materials.⁹⁰ These assessments also allow for a larger pool of comparison students since they are given across numerous states. Teachers will naturally focus most heavily on what is on the test. So if states adopt tests that are not Common Core aligned, the significant promise of the Common Core will be undermined.

Not all states are as far along as Kentucky. Each situation will require different interventions and policy solutions. But for states to experience similar student achievement gains, they must stay the course with the Common Core.

States and districts should provide teachers with additional, dedicated time and professional development opportunities

High-quality, ongoing, and readily available professional development allows teachers to internalize the standards with the help of effective instructors. States and districts also should develop a standards translation guide for teachers.

Incorporating conceptual mathematics into American math education simply cannot happen without teachers. No matter the standards or the quality of the curricula, students will only master the material if a teacher teaches it effectively. And since many of the teachers currently in the classroom were taught using procedural instructional approaches, the transition will take time. Teachers need plenty of opportunity to incorporate conceptual math into their instruction.

To manage this shift effectively, states and districts must provide ample time away from teaching for teachers to learn the standards and provide ongoing professional development to support them as they align their instructional practice with the standards. In fact, teachers will need more than the usual amount of professional

development to unpack the new standards and develop effective strategies to teach conceptual math. Furthermore, professional development also must include examples of student work so that teachers can practice evaluating against the standards.

Another benefit of the Common Core is that professional development can be shared within and across states. No longer must teachers wait until they receive district-sponsored professional development to work on their craft. Now, they can learn from teachers in neighboring district or from educators on the other coast. For example, the Oregon Department of Education has compiled a wide variety of Common Core professional learning resources on its website.⁹¹ The resources range from those developed in Oregon, as well as modules and resources from California, Rhode Island, nonprofits, and many other organizations. The American Federation of Teachers has also developed an online repository, Share My Lesson, for teachers to share Common Core-aligned lessons and instructional materials.⁹²

There are numerous other organizations that have developed and vetted Common Core-aligned instructional materials to share with schools and educators across the nation. The Mathematics Assessment Project, for example, provides hundreds of grade-specific mathematics lessons that teachers can use and adapt to meet their students' needs.⁹³ There are many other organizations that offer educators similar resources as well.

Finally, districts should develop a standards translation guide to help teachers refine their instruction in order to ensure that students meet the standards. Districts should adapt and build on existing work that interprets and applies the standards to classroom practice. For example, the Teaching Channel website includes numerous videos to help teachers work through the standards, including videos on how to unpack the standards, how to read the Common Core, and how to use incorrect math answers to support students learning conceptual math.⁹⁴

Districts should communicate regularly with parents and provide training and resources

Much of the controversy around Common Core math comes from parents who are frustrated, confused, or skeptical of the “new” math their children are learning. This anxiety likely stems from a few different problems. Some students are assigned homework questions that are poorly constructed, low quality, and not aligned with the Common Core. Some school districts have done a poor job communicating

with parents about the changes taking place in their children's math classes. Others have done a poor job of preparing teachers for the transition to the new standards, and educator anxiety has been telegraphed to children and parents. Taken together, examples of low-quality assignments and the lack of meaningful communication about the standards can lead some parents to believe that the Common Core is too complicated or does not make sense for their children.

For parents to support their children's learning, they must understand what is happening in the classroom. To do this, districts and schools should engage parents through thoughtful and consistent communication. In particular, as math educational practice changes somewhat under the Common Core, parents should be given opportunities to see how this approach to mathematics works; how it relates to math as they learned it themselves; and finally, how learning mathematics with conceptual understanding, as well as the standard algorithms, better prepares students for college and careers.

To engage with parents about conceptual math, many school districts across the country host math nights. At Freewill Elementary School in New York's Wayne Central School District, there was a parent math workshop during which parents worked with teachers on problems similar to the ones their children were studying.⁹⁵ Working directly with parents and showing them how conceptual math works helps parents familiarize themselves with math in the Common Core and better prepares them to work with their children.

There are also other resources available to help districts communicate with parents about the Common Core. Be A Learning Hero⁹⁶ provides rich resources for parents trying to help their kids with new Common Core-aligned content, as well as links to partner sites, such as Khan Academy, where parents can go for additional help. And the National Urban League has developed a suite of parent resources as a part of its campaign "Put Our Children 1st: Common Core for Common Goals," including short fact sheets that explain the background of the standards, what parents need to know, and how transitioning to the Common Core serves student interests.⁹⁷ There is also a guide that helps parents take action and ensure that schools are sufficiently prepared to teach the Common Core, have all the necessary resources, and have plans to inform parents on how to support their children's education at home.⁹⁸ Similarly, the National Parent Teacher Association offers a Common Core toolkit with parent guides on a wide range of topics.⁹⁹

Finally, for parents seeking to unpack particular standards or looking for example problems for each math standard, Inside Mathematics has an expansive suite of Common Core resources.¹⁰⁰ For example, under “mathematical practice standards,” parents and educators can see information and videos that explain each mathematics domain, including “reason abstractly & quantitatively,” unpacked by grade.¹⁰¹ Furthermore, the organization also offers grade-specific information, videos, and practice problems sorted by grade and specific content standard.¹⁰²

States and districts should review curricula and instructional materials to ensure that they are high-quality and aligned with and supportive of Common Core math

Kentucky’s successful implementation of the Common Core demonstrates that the curricula and materials teachers use play a huge role in student achievement. For instruction to be effective, educators must use aligned curricula and materials. Unfortunately, many textbooks and other materials are the same old resources repackaged and labeled as “Common Core-aligned.”¹⁰³ This disconnect creates numerous problems and undermines the goal of improved student achievement.

To avoid this problem, states and districts should collaborate with educators and other experts to review curricula and instructional materials to ensure that they are indeed aligned with the Common Core. Districts may choose to do this by creating their own materials; one example of this is in Marquardt School District 15 in Illinois. As early as 2011, Marquardt mathematics instructional specialists organized teachers to collaboratively build unit plans and other Common Core-aligned instructional materials. The resources were then implemented in schools, with these teachers serving as a support team.¹⁰⁴

But for districts that seek to rely on external expertise, there are a variety of resources and tools that nonprofit organizations have developed to help states and districts in this process. Achieve, a national education nonprofit, and the Council of Chief State School Officers developed a toolkit to help evaluate the alignment of instructional materials and assessments with the Common Core.¹⁰⁵ Using this toolkit, educators are able to determine whether the materials they are using do in fact help students meet the expectations of the Common Core.

Ed Reports, a recently created K-12 education nonprofit, assesses the alignment of instructional materials with the Common Core.¹⁰⁶ Commonly used textbooks and other curricular resources are evaluated for their alignment and usability across relevant grade spans. For example, the organization evaluated grades six through eight Eureka Math curricula and instructional materials and found that they are fully aligned for each grade but the materials only partially meet usability expectations.¹⁰⁷ All of the reports and findings are available on the nonprofit's website free of charge for states, districts, and teachers.

Teacher preparation programs should incorporate conceptual mathematics into curricula

Numerous recent studies have shown that teacher preparation programs could better prepare their graduates for the challenges of the classroom.¹⁰⁸ The states that have adopted the Common Core must ensure that prospective teachers are taught to provide high-quality mathematics instruction that includes conceptual math and aligns with the standards. Just as current teachers receive professional development on conceptual math instruction, future teachers must also learn the practice and pedagogy to ensure they are prepared to teach to the Common Core on their first day in the classroom.

In order to transform teacher preparation in mathematics, the Mathematics Teacher Education Partnership, or MTE-P, was formed in 2012 through the Association of Public and Land-Grant Universities.¹⁰⁹ The organization includes more than 100 universities and nearly 150 K-12 schools and districts in 30 states and prepares more than 8,000 teachers each year.¹¹⁰ To develop policy and to influence how teacher preparation programs teach math instruction aligned with the Common Core, the MTE-P formed Research Action Clusters to address specific needs of teacher preparation programs. The domains of work include improving first- and second-year mathematics courses; developing reliable and aligned math assessments; and creating impactful clinical experiences for prospective teachers.¹¹¹

The best practices and policies adopted from initiatives such as the MTE-P should be shared widely with institutions of higher education, state education agencies, and local school districts if they are to have the largest possible impact on mathematics instructional practice.

Conclusion

The expectations of today's workforce have changed dramatically and are far more demanding than ever before. To prepare graduates to be ready to meet these challenges and compete successfully for high-quality jobs—particularly those in STEM fields—mathematics education must become similarly rigorous and challenging.

The Common Core helps states and districts raise the bar for all students by incorporating conceptual math directly into state academic standards. This evolution marks a critical addition to how students are taught and learn math. Rather than only learning mathematical processes, students are first taught the underlying concepts, values, and numerical relationships involved. In many instances, this approach can create opportunities for students to discover more advanced concepts and relationships, such as the fundamental principles of fractions as they work with shapes in first grade.

Conceptual mathematics also has a considerable foundation in research, which demonstrates clearly that American students have poor numeracy knowledge and skills and that learning math conceptually and then practicing it through the traditional algorithmic approach is an effective way to improve.

Effectively teaching the Common Core and conceptual mathematics is challenging, and it will require considerable patience and investments from states and districts to support educators teaching to these more challenging standards. But the stakes are high: The performance of American students continues to slip and lag behind their peers around the world, and simply put, failing to improve the nation's math education would harm students' opportunities and stifle the economy.

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